Explicit time and space dependence of ballistic molecular heat pulses

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(Received 2 April 1998)

A kinetic description of ballistic molecular heat pulses is presented. By solving the Boltzmann equation containing a source term we study the explicit time and space dependence of these pulses for various types of sources and detectors. Assuming that in the case of ^{4}He the source consists of a pointlike source that radiates Maxwellian pulses, the agreement with experimental findings is excellent for point or extended particle detectors. In the case of ^{3}He beams one needs to introduce two sources of different temperatures. This may be attributed to the enormous difference of heat conductivity of ^{3}He and superfluid ^{4}He films.
[S1063-651X(98)03309-1]

PACS number(s): 05.20.Dd, 44.90.+c, 51.10.+y, 66.70.+f

I. INTRODUCTION

The heat pulse experimental technique has been applied for studies of the velocity spectrum of atoms evaporated from a free liquid ^{3}He and ^{4}He surface by Andres and coworkers [1,2]. The authors concluded that this spectrum is Maxwell-Boltzmann-like in nature.

In this paper we present the kinetic description of such pulses in the ballistic (collisionless) regime. With the help of the source term technique [3], for various types of sources we obtain the one-particle distribution function, which allows us to derive the formula for the energy density current. This technique we successfully applied to the problem of the kinetic description of heat phonon pulses and to study the phonon focusing phenomenon [4,5].

The expressions obtained for the energy density current are used to calculate the shapes of signals radiated by different sources, propagating in gaseous ^{3}He and ^{4}He of low density and registered by the bolometers of various geometrical forms.

II. KINETIC DESCRIPTION OF MOLECULAR HEAT PULSES

Consider a gas of classical particles each of mass m, momentum $\mathbf{p}=m\mathbf{v}$, and energy $\varepsilon(v)=mv^2/2$). Their group velocity $\mathbf{v}_g=\partial\varepsilon/\partial\mathbf{p}$ is equal to the particle velocity $\mathbf{v}$. The state of a rarefied gas of particles or quasiparticles is described by the distribution function f. For a system of classical particles the distribution function depends on the group velocity $\mathbf{v}_g$, the space variable $\mathbf{r}$, and time t.

The helium introduced to the chamber forms a condensed film over all surfaces (see Fig. 1). For a finite source of dimensions 3.7×3.7 mm^2 and for a helium film thickness of 200 Å the total number of atoms is $N_f\sim10^{15}$. In the experiments discussed nonequilibrium beams of helium particles were generated using metallic films heated by pulses of current, the power of which is controlled.

In agreement with Andres *et al.* [1], we assume that immediately after evaporation atoms fill a region of volume V^* and there is a strong interaction of these particles resulting in quick thermalization. Denote the number of particles filling V^* by N_b ($N_b\leqslant N_f$). The gas of these evaporated particles attains a temperature T^* different from the vapor (ambient) temperature T. In the case of the purely ballistic motion of pulses we assume $T=0$ K. In the experiments considered one measures the velocity spectrum of this gas.

Suppose further that the time of formation of each molecular heat pulse is much shorter than the mean time lag after which pulses arrive at the detector. The last assumption concerning pulses is that, in agreement with Cole [6] and experimental findings [1,2], they are Maxwellian with temperature T^*, i.e., the distribution of coordinates and velocities of the pulse particles is given by the familiar Maxwell distribution function

$$f_m(v,T^*)=\frac{1}{V^*}\left(\frac{\gamma}{\pi}\right)^{3/2}\exp(-\gamma v^2), \tag{1}$$

where

$$\gamma=\frac{m}{2k_BT^*}.$$

Consider a small volume d^3r surrounding the point $\mathbf{r}$ and the small volume d^3v in the velocity space containing the point $\mathbf{v}$. The product $f_m(v,T^*)d^3rd^3v$ is the average number of particles in d^3r with velocities from d^3v (i.e., this product is a dimensionless quantity $[f_m(v,T^*)d^3rd^3v]=1$).

Let us recall that the thermal energy of the beam $E_{\rm th}(T^*)$ is equal to $N_b(\frac{3}{2}k_BT^*)$. Similarly, the total momentum of the beam $P_{\rm th}$ also depends on T^*, namely, $P_{\rm th}(T^*)=2m(2k_BT^*/m\pi)^{1/2}N_b$.

Assume that the z axis of the Cartesian coordinate system is perpendicular to the surface of the film and is directed towards the detector. The remaining axes are lying in the plane of the source and are arbitrarily oriented. The detector of the pulses (a superconducting bolometer sensitive to the energy carried by the particles of the pulses) and the source were mounted opposite each other at a distance L. Only particles with a positive z component of the group velocity v_z may reach the detector.

In the case of experiments on the propagation of heat pulses, the distribution function obeys the kinetic Boltzmann equation with the collision integral term $C[f]$ (which ac-

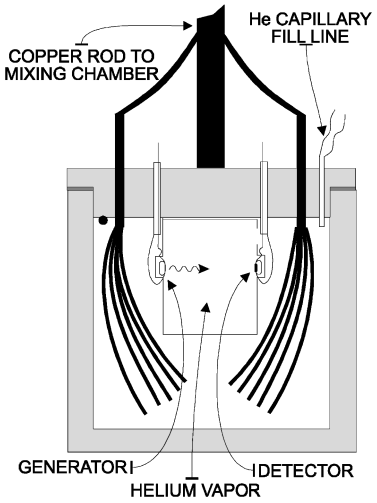

FIG. 1. Scheme of the helium chamber used in pulsed molecular beam experiments.

counts for all scattering processes suffered by the particles of a beam) and the source $\dot{\zeta}(\mathbf{v};\mathbf{r},t)$ term,

$$\frac{\partial f(\mathbf{v};\mathbf{r},t)}{\partial t}+\mathbf{v}\boldsymbol{\nabla}f(\mathbf{v};\mathbf{r},t)=C[f(\mathbf{v};\mathbf{r},t)]+\dot{\zeta}(\mathbf{v};\mathbf{r},t). \quad (2)$$

In the ballistic regime one can discard the collision term $C[f]$. Of course, this term plays an important role in the description of processes of thermalization because collisions with thermal vapor particles provide the crossover from the ballistic to the hydrodynamic (i.e., collision dominated) regime of propagation of molecular heat pulses observed by Andres and co-workers [1,2]. Such a crossover from the ballistic to the collision dominated regime of motion can be studied for the Lorentz gas models [7,8] and for phonon heat pulses propagating in elastic isotropic media containing point mass defects [9].

Consider the source term in the form

$$\dot{\zeta}_G(\mathbf{v};\mathbf{r},t)=\delta(\mathbf{r})\,\delta(t). \quad (3)$$

The solution of the collisionless Boltzmann equation (CBE) with such a source term, called the Green's function, reads

$$f_G(\mathbf{v};\mathbf{r},t)=\theta(t)\,\delta(\mathbf{v}t-\mathbf{r}). \quad (4)$$

Here $\theta(x)$ is the Heaviside step function and $\delta(x)=d\theta(x)/dx$ is the Dirac delta distribution. Since x, y, and z are components of the radius vector $\mathbf{r}$, the vectorial δ function is the product of three δ functions of the scalar argument $\delta(\mathbf{r})=\delta(x)\,\delta(y)\,\delta(z)$. Recall that $[\delta(\mathbf{r})]=l^{-3}$, where l is the length.

The solution of the CBE for an arbitrary source term $\dot{\zeta}(\mathbf{v};\mathbf{r},t)$ can be obtained with the help of the Green's function [3], namely,

$$f_{\dot{\zeta}}(\mathbf{v};\mathbf{r},t)=\int d^3r'\int_{-\infty}^{+\infty}dt'\,\dot{\zeta}(\mathbf{v};\mathbf{r}',t')f_G(\mathbf{v};\mathbf{r}-\mathbf{r}',t-t'). \quad (5)$$

On this solution we impose the natural condition [10]

$$\int_{V^*}d^3r\int_{-\infty}^{+\infty}dt\int d^3v\;\dot{\zeta}(\mathbf{v};\mathbf{r},t)\varepsilon(v)=E_{\mathrm{th}}(T^*). \quad (6)$$

III. THE DISTRIBUTION FUNCTION FOR SELECTED KINDS OF SOURCE TERMS

In the experiments discussed [1,2] the linear distance L between the source and detector planes was comparable to the linear dimensions of the source and detector. This means that the geometry of the experiment was intermediate between a pointlike and an infinite slab. Therefore, we considered several types of sources.

A. A point source

For the described experimental conditions for a point source located at $\mathbf{r}=0$ generating very short pulses, the source term $\dot{\zeta}_p(\mathbf{v};\mathbf{r},t)$ has the form

$$\dot{\zeta}_p(\mathbf{v};\mathbf{r},t)=A_pV^*\theta(v_z)f_m(v,T^*)\,\delta(t)\,\delta(\mathbf{r}), \quad (7)$$

where A_p is a constant. One may easily check that for $[A_p]=1$ the source term has the correct dimension, namely, $[\dot{\zeta}_p(\mathbf{v};\mathbf{r},t)d^3\mathbf{r}d^3v]=t^{-1}$.

The constant A_p is determined from the condition (6). Calculating integrals, we find $A_p=2N_b$. Hence the solution of the Boltzmann equation with the source term (7) reads

$$f_p(\mathbf{v};\mathbf{r},t)=2N_bV^*\theta(v_z)f_m(v,T^*)\,\theta(t)\,\delta(\mathbf{v}t-\mathbf{r}). \quad (8)$$

B. A homogeneous finite source

Assume that atoms are evaporated by a homogeneous source in the form of a circle of radius ρ_s located on the plane $z=0$ with the center at $\rho=0$, where z and ρ are two of three cylindric coordinates. The third coordinate is the angle ϕ_ρ. As previously, if we assume that the duration of these pulses is very short, then

$$\dot{\zeta}_c(\mathbf{v};\mathbf{r},t)=A_c\theta(\rho_s-\rho)\,\delta(t)\,\delta(z)\,\theta(v_z)f_m(\varepsilon,T^*). \quad (9)$$

Calculating integrals in Eq. (5), we obtain the solution obeying the condition (6),

$$f_c(\mathbf{v};\rho,z,t)=\frac{2N_bV^*}{\pi\rho_s^2}\theta(v_z)f_m(\varepsilon,T^*)\,\theta(t)\times\delta(z-v_zt)\,\theta(\rho_s+v_\rho t-\rho), \quad (10)$$

where v_ρ, v_z, and ϕ_v define the velocity vector in the cylindric coordinate system.

C. Slab geometry

For an infinite slab the source term depends only on the z variable and time t,

$$\dot{\zeta}_\infty(\mathbf{v};\mathbf{r},t)=A_\infty\theta(v_z)f_m(\varepsilon,T^*)\,\delta(t)\,\delta(z); \quad (11)$$

hence the corresponding distribution function reads

$$f_\infty(\mathbf{v};z,t)=A_\infty\theta(v_z)f_m(\varepsilon,T^*)\,\theta(t)\,\delta(v_zt-z).$$

D. Density of energy current

The quantity measured in experiments on molecular heat pulses is the power density (density of energy per unit time) falling onto the surface with the normal $\hat{\mathbf{n}}$ at the point $\mathbf{r}$,

$$e^{(\dot{\zeta})}_{\hat{\mathbf{n}}}(\mathbf{r},t)\equiv\hat{\mathbf{n}}\mathbf{j}^{(\dot{\zeta})}_{E}(\mathbf{r},t)=\int d^3v(\hat{\mathbf{n}}\mathbf{v})\varepsilon(v)f_{\dot{\zeta}}(\mathbf{v};\mathbf{r},t). \quad (12)$$

In the agreement with Andres *et al.* [1], in our calculations we set $\hat{\mathbf{n}}=\hat{\mathbf{z}}$. Later we shall omit the index $\hat{\mathbf{z}}$ in $e^{(\dot{\zeta})}_{\hat{\mathbf{z}}}$.

For the fixed $\mathbf{r}$ the maximum of a pulse arrives at time $t_{\max}$. We denote the maximal value of the energy density by $e^{(\dot{\zeta})}_{\max}(\mathbf{r})$ [$e^{(\dot{\zeta})}_{\max}(\mathbf{r})=e_{\dot{\zeta}}(\mathbf{r},t=t_{\max})$]. Andres *et al.* [1] measured the ratio

$$I^{(\dot{\zeta})}(\mathbf{r},t)\equiv\frac{e_{\dot{\zeta}}(\mathbf{r},t)}{e^{(\dot{\zeta})}_{\max}(\mathbf{r})}. \quad (13)$$

Therefore, with the exception of the discussion of a solution for a point source, we shall confine ourselves only to this quantity normalized to unity, which we shall call the signal of the bolometer.

E. Current densities for a point source

We introduce the length $r=|\mathbf{r}|$ and the unit direction vector $\hat{\mathbf{r}}$ as well as two polar angles θ_r,ϕ_r. By introducing also the polar coordinates in v space $v=|\mathbf{v}|,\theta_v,\phi_v$ and by using the familiar identity

$$\delta(\mathbf{v}t-\mathbf{r})=[(vt)^2|\sin\ \theta_v|]^{-1}\delta(\theta_v-\theta_r)\,\delta(\phi_v-\phi_r)\,\delta(\mathbf{v}t-\mathbf{r}), \quad (14)$$

we get

$$e_p(\mathbf{r},t)=\frac{N_b m}{\pi^{3/2}}\theta(t)\gamma^{3/2}\frac{r^3}{t^6}\exp\left(-\gamma\frac{r^2}{t^2}\right)(\hat{\mathbf{z}}\hat{\mathbf{r}}). \quad (15)$$

The physical dimension of $e_p(\mathbf{r},t)$ is m/t^3 ($[e^{(\dot{\zeta})}(\mathbf{r},t)d^3v]=[\varepsilon/l^2t]=m/t^3$).

Similarly, we calculate an η component of the momentum density current tensor $\tilde{\Pi}$ $(\mathbf{r},t)$ per unit time falling onto the surface with the normal $\hat{\mathbf{z}}$ at the point $\mathbf{r}$,

$$p^{(p)}_{\eta}(\mathbf{r},t)=\sum_{\alpha,\beta=1}^{3}\hat{\eta}_{\alpha}\tilde{\Pi}_{\alpha\beta}(\mathbf{r},t)\hat{z}_{\beta}=\int d^3v(\hat{\boldsymbol{\eta}}\mathbf{v})(\hat{\mathbf{z}}\mathbf{v})f(\mathbf{v};\mathbf{r},t)$$

$$=\frac{2N_b m}{\pi^{3/2}}\theta(t)\gamma^{3/2}\frac{r^2}{t^5}\exp\left(-\gamma\frac{r^2}{t^2}\right)(\hat{\boldsymbol{\eta}}\hat{\mathbf{r}})(\hat{\mathbf{z}}\hat{\mathbf{r}}). \quad (16)$$

The momentum density is proportional to the sound pressure p (cf. [11,12]). Hence, for molecular pulses generated by a point source and propagating in this regime we obtain

$$p\sim\frac{z^2}{t^5}\exp\left(-\gamma\frac{r^2}{t^2}\right).$$

Note that for waves propagating in the same regime along the z axis Cook *et al.* have calculated the excess of stress $\delta\tilde{T}_{zz}(\mathbf{r},t)$ at a transducer [13] (cf. also [14,15])

$$\delta\tilde{T}_{zz}(\mathbf{r},t)\sim\frac{z^2}{t^3}\exp\left(-\gamma\frac{r^2}{t^2}\right). \quad (17)$$

Mayer and Sessler [15] considered the momentum density for collisionless propagation of sound in gases.

Consider the energy density and the density of the η-component of momentum falling onto the surface with the normal $\hat{\mathbf{z}}$ at the point $\mathbf{r}$ of the detector plane. Calculating the integral over time we get

$$e(\mathbf{r})=\int_0^{\infty}dt\ e_p(\mathbf{r},t)=\frac{E_{\rm th}(T^*)}{\pi r^2}(\hat{\mathbf{z}}\hat{\mathbf{r}}), \quad (18)$$

$$p^{(p)}_{\eta}(\mathbf{r})=\int_0^{\infty}dt\ p^{(p)}_{\eta}(\mathbf{r},t)=\frac{P_{\rm th}(T^*)}{\pi r^2}(\hat{\boldsymbol{\eta}}\hat{\mathbf{r}})(\hat{\mathbf{z}}\hat{\mathbf{r}}). \quad (19)$$

Thus we conclude that these quantities obey the Gauss-Lambert law valid for isotropic spaces [5]. Further on we confine ourselves to the case of energy pulses.

F. Energy current density for a homogeneous finite source

For a homogeneous finite source the bolometer signal at the point with cylindrical coordinates (ρ,ϕ_ρ,z) depends on the exponential function

$$I_c(z,\rho,t)=\theta(t)\gamma^{-1/2}\frac{z}{t^2}\left(1+\gamma\frac{z^2+u^2}{t^2}\right)\exp\left(-\gamma\frac{z^2+u^2}{t^2}\right),$$

where

$$u=\begin{cases}0 & \text{for } 0\leqslant\rho\leqslant\rho_s\\ \rho-\rho_s & \text{for } \rho>\rho_s.\end{cases}$$

G. Energy current density for an infinite slab

For an infinite slab we obtain

$$I_{\infty}(z,t)=\gamma^{-1/2}\frac{z}{t^2}\left(1+\gamma\frac{z^2}{t^2}\right)\exp\left(-\gamma\frac{z^2}{t^2}\right). \quad (20)$$

IV. COMPARISON WITH EXPERIMENTAL FINDINGS

Using the experimental data of Andres *et al.* [1] (their Figs. 2–5) and the equation of the state of the ideal gas, we estimated the mean free path l of beams of helium particles. We used the standard formula for it

$$l=(\sqrt{2}\pi nd)^{-1},$$

where n is the density of helium vapor and d is the diameter of particles ($d\simeq 2.2$ Å). In this way we get $l_<$, the lower bound of l. For ^{4}He $l_<=15.4$ mm, whereas for ^{3}He $l_<=8.19$ mm. Since the distance between the source and detector is equal to 2.34 mm we conclude that the scattering of

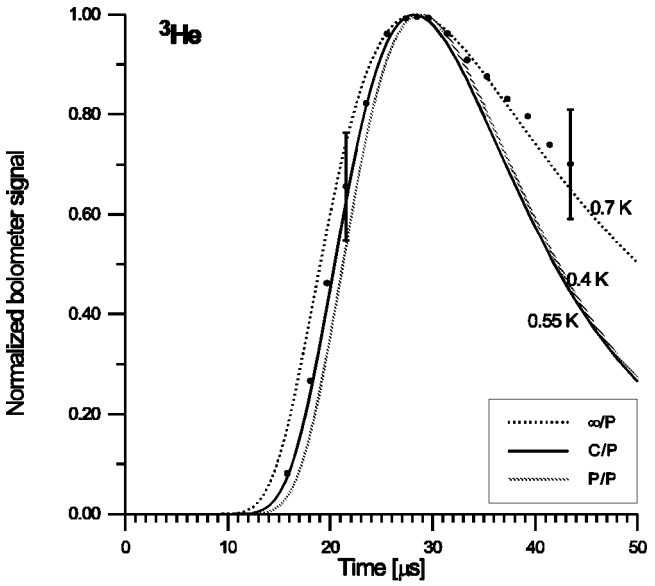

FIG. 2. Calculated bolometer signal vs time for ^{3}He. The temperature estimated by Andres *et al.* [1] is 0.6 K. Dots represent experimental results. Typical error bars are shown.

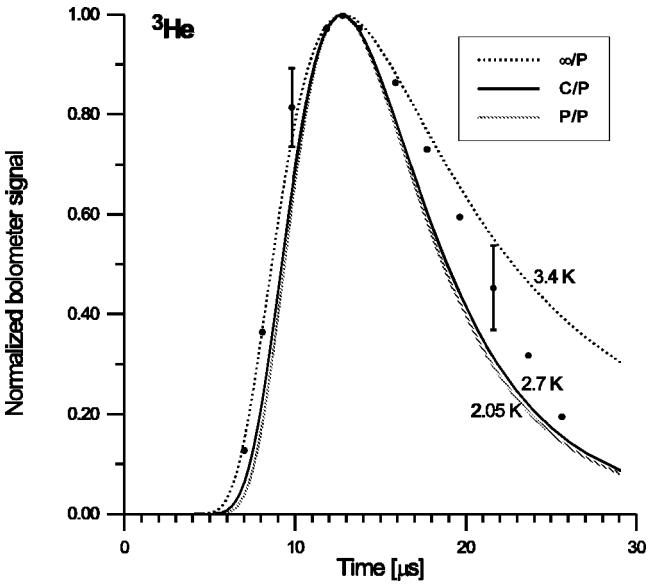

FIG. 3. Calculated bolometer signal vs time for ^{3}He. The temperature estimated by Andres *et al.* [1] is 2.2 K. Dots represent experimental results. Typical error bars are shown.

the beam particle by vapor particles cannot influence the form of pulses. So one may study the role of the form of sources only.

A. Molecular ^{4}He beams

Each pulse is characterized by two times, namely, by the arrival time t_{arr}

$$I(t,z=L)=\begin{cases}0 & \text{for } t<t_{\mathrm{arr}}\\ I(t) & \text{for } t>t_{\mathrm{arr}},\end{cases}$$

and the time of arrival of the maximum of a pulse t_{max}. The remaining characteristic is the pulse temperature T^*, which we treat as a parameter that should be fitted.

The form of pulses depends not only on the type of source but also on the particular detector. We considered a pointlike detector, a finite detector having the form of a circle, and a detector covering the whole plane.

Generally, for all types of sources and detectors there exists a time interval $(0,t_{\mathrm{arr}})$ for which the amplitude of signals is negligibly small. Additionally, all values of the arrival time obtained here are rather close to those observed in experiments [1]. However, the characteristic times t_{arr} and t_{max} and shapes of signal, as well as the fitted temperatures, are closest to the experimental findings [1] for (i) the point source and point detector (which we denote p/p), (ii) the point source and a circular detector with area of 3.7×3.7 mm^2 (c/p), and (iii) the point source and a detector covering whole plane $z=L$ (∞/p). The results obtained are presented in Figs. 2–5 and Table I.

The results obtained suggest that the sources used in experiments [1] behave as sets of point-like sources. However, inspecting Figs. 2–5 one sees that for ^{4}He fits are much better than for ^{3}He, which we may attribute to an enormously large heat conductivity of superfluid ^{4}He films in comparison to ^{3}He films. Therefore, one can expect that the latter films are less homogeneously heated than the former. We check this observation in Sec. IV B.

Figures 2–5 are labeled with the kind of helium atoms. On each of them there are three curves corresponding to the three above-mentioned configurations for which we obtained the best agreement with the experimental results. One should notice that the curves on Figs. 2–5 corresponds to different fitted temperatures T^*.

B. Molecular ^{3}He beams

For simplicity we model inhomogeneously heated helium films in only two ways. The first is a homogeneously heated circle with the center at $\rho=z=0$ and a superimposed point

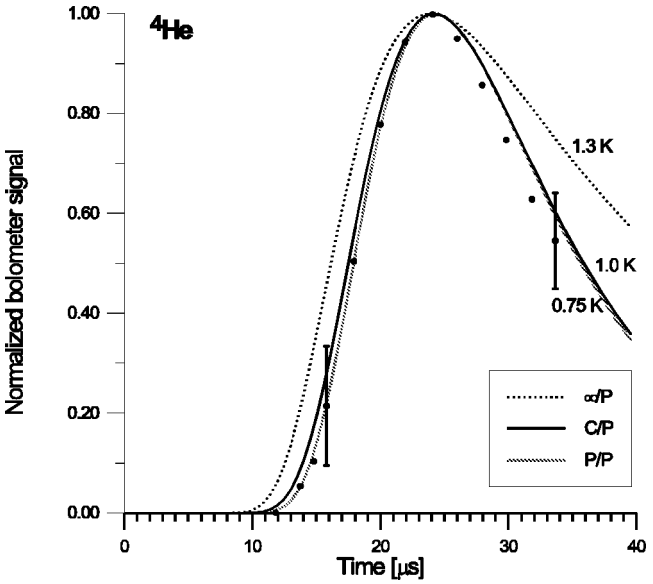

FIG. 4. Calculated bolometer signal vs time for ^{4}He. The temperature estimated by Andres *et al.* [1] is 1.1 K. Dots represent experimental results. Typical error bars are shown.

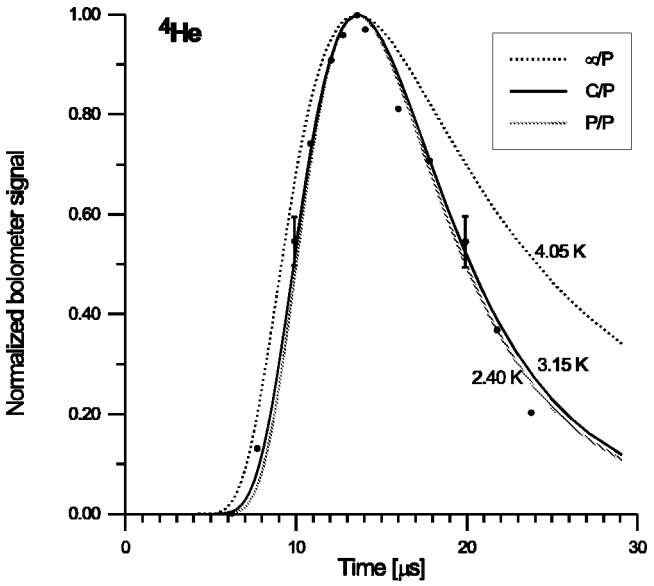

FIG. 5. Calculated bolometer signal vs time for ^{4}He. The temperature estimated by Andres *et al.* [1] is 2.9 K. Dots represent experimental results. Typical error bars are shown.

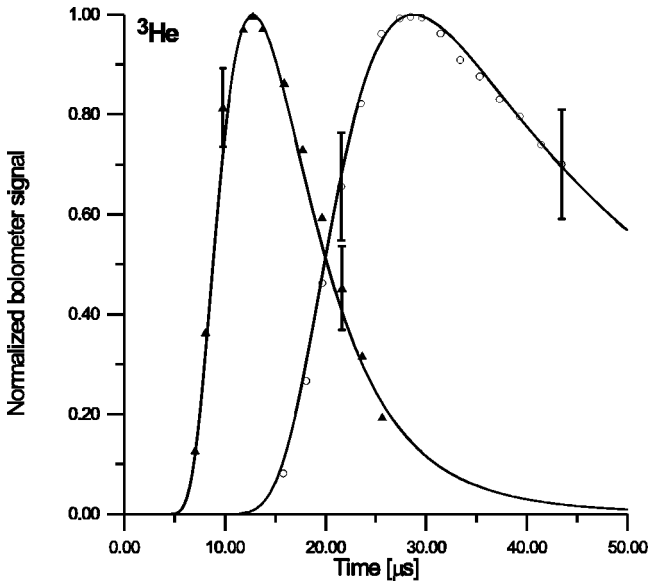

FIG. 6. Calculated bolometer signal vs time for a combined source of ^{3}He beams that consists of (i) a heated circle and a superimposed point source (the heating pulse peak power is 0.018 ergs) (open circles) and (ii) two point sources generating two beams of different temperatures (the heating pulse peak power is 1.8 ergs) (triangles).

source (a ''boiling spot'') located in the center of this circle. They both radiate Maxwellian beams of temperature T_c^* and T_p^*, respectively. The second kind of inhomogeneous source consists of two point sources generating two Maxwellian beams of temperatures T_1^*, T_2^*.

We found that for a colder ^{3}He film the fit is excellent for the first kind of source with $T_c^* = 0.25$ K and $T_p^* = 0.6$ K (Fig. 6, circles). The second kind of source (two boiling spots) is better suited for a more heated ^{3}He film. The fit is best for $T_1^* = 1.58$ K and $T_2^* = 3.16$ K (Fig. 6, triangles). In both cases we assume that the detector has the form of a circle.

V. CONCLUSIONS

Using the technique of source terms added to the Boltzmann equation we have studied the shapes of molecular pulses in the collisionless regime of propagation. We have shown that, in agreement with Andres *et al.* [1], the assumption that the pulses are Maxwellian is reasonable and its form provides information about the state of films that evaporate beams of particles. Since we only had at our disposal experimental results in the form of plots, we considered the simplest models of sources. More complicated models with many fit parameters (e.g., coordinates of numerous sources) are beyond the accuracy of the experimental data used. Our approach is semiphenomenological and certainly the construction of models used should be based on a deeper understanding of the underlying physical processes. However, even such a simple approach seems to be quite successful; therefore, we expect that our results may shed some light on the problem of the heat exchange between cooled specimens and a helium bath.

TABLE I. Fitted temperatures of pulses found in Andres, Dynes, and Naranayamurti's experiments and calculated here for different types of source-detector configurations.

Configuration	Pulse temperature T^* (K) ^{3}He		^{4}He	
Andres *et al.*	0.60	2.20	1.10	2.90
p/p	0.40	2.05	0.75	2.40
c/p	0.55	2.70	1.00	3.15
∞/p	0.70	3.40	1.30	4.05
circle/circle	0.90	4.50	1.70	5.30
slab geometry	0.75	3.75	1.40	4.45

ACKNOWLEDGMENTS

We would like to thank Venky Narayanamurti for his valuable comments and a critical reading of the manuscript. In particular, he drew our attention to the role of the enormously large difference in heat conductivities of superfluid ^{4}He and normal ^{3}He films in the process of the formation of pulses.

APPENDIX: PHONON PULSES IN ISOTROPIC ELASTIC MEDIA

For comparison, consider acoustic phonon pulses propagating in isotropic media [9]. For such media the phonon energy ε is a linear function of the length p of quasimomentum $\mathbf{p}$, i.e., it obeys the linear dispersion relation

$$\varepsilon_j(\mathbf{p}) = c_j p \quad (j = 0,1,2),$$

where $p = \hbar k$, $k = |\mathbf{k}|$ being the length of the wave vector $\mathbf{k}$ of an acoustic phonon of polarization j ($j = 0$ for longitudinal, $j = 1$ for fast transverse, and $j = 2$ for slow transverse phonons), and c_j is the corresponding phase velocity. The group velocity of acoustic phonon wave packets moving in isotropic media is

$$\mathbf{v}_g = c_j \hat{\mathbf{k}},$$

where $\hat{\mathbf{k}} = \mathbf{k}/k$ is the direction of the phonon wave vector.

Consider a pointlike Planckian source producing short pulses of acoustic phonons of temperature T^*,

$$\dot{\zeta}(\mathbf{k}, j; \mathbf{r}, t) = AV^* f_{\mathrm{pl}}\left(\frac{\varepsilon}{k_B T^*}\right) \theta(\hat{k}_z)\, \delta(\mathbf{r})\, \delta(t),$$

where $f_{pl}(\varepsilon/k_B T^*) = [\exp(\varepsilon/k_B T^*) - 1]^{-1}$ is the Planck distribution function and V^* is the volume of the source, a heated region of a crystal or metallic film. The distribution function reads

$$f(\mathbf{k}, j; \mathbf{r}, t) = AV^* f_{\mathrm{pl}}\left(\frac{\varepsilon}{k_B T^*}\right) \theta(\hat{k}_z)\, \delta(\mathbf{r} - c_j \hat{\mathbf{k}} t)\, \theta(t).$$

A condition similar to Eq. (6) allows one to relate the constant A to the suitable part of thermal energy of the source injected into the specimen $E_{\mathrm{th}}^{(s)}(T^*)$ [10]. The power density $e(\mathbf{r}, t)$ falling onto the bolometer surface with the normal $\hat{\mathbf{n}} = \hat{\mathbf{z}}$ can be easily calculated

$$e(\mathbf{r}, t) \equiv j_z^E(\mathbf{r}, t) = \frac{E_{\mathrm{th}}^{(s)}(T^*)}{2\pi r^2} \frac{1}{3} \sum_{j=0}^{2} \left(\frac{c_D}{c_j}\right)^3 \delta\left(t - \frac{r}{c_j}\right),$$

where c_D is the Debye velocity of considered isotropic medium

$$c_D^{-3} = \tfrac{1}{3} \sum_{j=0}^{2} c_j^{-3}.$$

We see that other than in the case of molecular pulses, phonon pulses have the form of very narrow maxima traveling with the corresponding group velocities.

In the case of *perfect anisotropic* media [4] the complicated nonspherical shapes of phonon surfaces of constant energy result in an increase of the number of observed pulses because one ought to account for contributions of all points of this surface at which the normal, i.e., the group velocity vector, is parallel to the vector $\mathbf{r}$ joining a point source with a point detector. In addition, such surfaces make pulse amplitudes dependent on the direction of propagation (phonon focusing) and broaden them. However, unlike molecular pulses, such broadened maxima still move with the corresponding group velocities.

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